Mathematical Thinkers Like Me

A. Innovative Approach

Need

Students who are Black, Latinx, Native American, and those identifying with other groups historically marginalized in the US disproportionately shoulder the impact of what Gloria Ladson-Billings named the "education debt.[1]" This debt has accumulated through policies, practices, and educational structures shaped by prejudiced, elite-oriented, misinformed, and often racist attitudes[2]. Shamefully, for students in marginalized groups such as these, there has been inequitable access to opportunities to successfully learn and this includes development of executive functions.

Another consequence of this educational inequity is what Rochelle Gutierrez calls dehumanization in math education. Given that interest[3] and cultural relevance[4] are primary factors governing learning and the use of what is learned, too many students, particularly students of color, do not recognize themselves in the mathematics they are learning. In response to marginalization and to achieve rehumanization, we must center student voices in how we attend to, feature, respond to, recognize the authority of, and share power with our students of color (and their teachers)[5].

It is common for schools to have low expectations for students of color and not high-quality mathematics, which then leads to a vicious cycle: students lose interest; they see math as for others not like them; they stop seeking math knowledge and challenging themselves; capacities such as executive functions are not exercised and not supported with persistence; and now, as the mathematics curriculum advances, these students are less able to engage it[6].

High-quality math experiences feed interest and provide the richness and complexity that challenge the strengthening of strategic practices, such as executive functions and mathematical habits of mind that are all necessary for further learning and success in math. As students grow in these ways, the success and sense of increased capacity contribute to identities that include mathematical competence. Set these experiences in a context where students feel valued and recognized for the many ways that they participate in mathematical thinking and these students will appear just as capable as any other.

In light of the above, we see that the three foci of EF+Math (conceptual understanding in math, executive functions, and equity) work tightly together and depend on each other for their advancement. One of the suggestions emerging from syntheses of research about executive functions (EF) interventions is that EF interventions should focus on implicit development in embedded tasks.[7] Narrowly defined laboratory tasks do not contain the richness that challenges executive functions, nor do these foster the interest needed to persist and seek continual challenge[8]. There is too large a gap for transfer. Many programs perpetuate the accumulation of debt since these students are deprived of the rich, interdependent environment needed for their capacities to develop[9].

Strategy

We will collaborate with teachers and students to build Mathematical Thinkers Like Me (MLM), a program centered around online collaborative problem solving that supports student storytelling and community sharing about their ongoing journey as mathematical thinkers. Student voice is at the center of MLM's educational process, focusing on student success with rich, high-quality mathematics; cultivating their interest and mathematical identities and strengthening their sense of control in their learning and the pursuit of their interests.

MLM's educational process is informed by pedagogical and organizational practices drawn from three instructional models: culturally responsive teaching, complex instruction, and challenge-based learning. The use of these models be shaped and guided by a co-design process that establishes mutual ownership and brings the insight and effort of all participants to the success of MLM.

The MLM project is guided by a multi-layered Theory of Action. Two central ideas throughout the layers are interdependence and interest. Interdependence in the sense that there is no one activity, or outcome, or cause, or mechanism that works in isolation. Interest plays a central role as both outcome and causal mechanism in the projects theories of action. Here, at a general level, is the MLM overarching Theory of Action:

Activities: Students collaborate online to solve rich, challenging mathematical tasks, and then use these experiences to tell evidence-based stories about their development as mathematical thinkers, and share these stories with others with whom they can identify.

Outputs: Motivated engagement in math problem solving, more developed collaborative practices, increased opportunities to see oneself as a valued contributor to math problem solving, and a learning environment that provides: support and modeling in doing math; metacognition about one's learning, executive functions, and experience of math; and the experience of commonalities and differences in a community of students.

Outcomes: Increased control of one's participation in learning math (EF), more developed mathematical practices, expanded and maintained interest in math, developing identities and self-efficacy as a mathematical thinker.

Impact: Students of color experiencing more success with rich mathematics, having stronger executive functions, more developed conceptual understanding, and increased likelihood of pursuit of math-informed careers.

Causal links and mechanisms: The opportunity to connect with other students is an initial trigger for interest, as are the novelty and features of the dynamic math environment. Interest is then sustained through collaborative problem solving and storytelling. In contrast to the often individualized approach to math learning, the collaborative experience can match the community approach that many students of color, experience in their lives. The various roles in collaboration and the use of storytelling expand the modes in which students can participate, appealing to different areas of competence. The interest development and rehumanization supported by culturally responsive pedagogy fosters multiple cognitive and affective connections leading to stronger meaning-making, retention, and resilience in the face of cognitive load[10][11].

As interest increases, it supports the persistence and the generation of new challenges that are required for the development and maintenance of executive functions[8]. Executive functions become focal and accessible by connecting them to the collaborative practices that support success in math problem solving and learning. MLM challenges students to get good at collaborative practices and to tell stories about them—practices such as turn-taking and balancing participation, holding on to one's activated knowledge and questions while attending to the work of others in the math space, and engaging with the different solution strategies and observations of one's collaborators. We call these Executive Functions in Practice (EFP) because of the way in which they map directly to inhibitory control, working memory, and cognitive flexibility. The development of EF is implicit and emergent, embedded in the regular new routines of collaborative math learning, which then increases the rate of practice and the nearness of transfer to other mathematical contexts. Finally, interest is freer to develop as students experience success and develop a sense that they and others like them are mathematical thinkers, which changes their cognitive stance from disconnection and avoidance to openness and seeking behaviors[12].

A collaborative online dynamic environment such as Vitural Math Team (VMT) has particular affordances that facilitate these processes. The collaborative space is purposely restricted to just the dynamic math space and a text chat, focusing participants on the mathematics, enabling individuals to concentrate on their own thoughts while also taking in the math work being done, without all of the other communication and activity that would happen with video and audio. Only one person at a time can manipulate the central dynamic math area, and everyone is able to chat while this is happening, which provides some support for students regulating their participation.

This takes place through a digital device (tablet or laptop computer), the sessions are recorded and can be replayed. This makes it possible for teachers to focus on mathematical thinking and help students develop their mathematical practices. Typically, teachers and researchers do not have access to students' problem-solving process but only to the end products or intermediate moments. Even though educators know that mathematical practices are as important as mathematical content knowledge, textbooks and classroom instruction are still mostly focused on math content[13]. How many students feel some version of "I don't even know how you knew to think about that or what to pay attention to!"? The collaborative context provides learners with access to the process of thinking and models from others to draw on. For teachers, MLM will enable them to review at any time, the thinking and work of a group or student and gain insight into what would help them develop their mathematical practices.

Teachers can greatly increase the value of the experience by balancing students' attention to the different elements that work together for learning: math content knowledge, mathematical practices, collaborative practices, negotiating roles and status in participation, and relevance. In this context the MLM teaching approach draws from instructional practices that are designed for these situations:

- Complex Instruction (CI), an instructional approach in which cooperative group support access and equitable relations, as well as rigorous work with disciplinary content (e.g. mathematics) in academically and linguistically heterogeneous classrooms[14][15]. CI learning tasks are open-ended, both in how students arrive at solutions and the identified solutions.
- Challenge-based Learning (also known as Productive Failure) combines learning through problem solving with direct instruction to optimize student success[16][17]; and
- Culturally Responsive Teaching (CRT), an instructional approach that (re)humanizes mathematics learning througs making learning more accessible, relevant and meaningful by using students' local knowledge and sociocultural experiences to support engagement with abstract mathematical ideas[18][19].

Using these three approaches, teachers coordinate and optimize the MLM tasks, similar to an orchestral conductor, ensuring the approaches work together to support students' interest and identity development that drives the desired outcomes.

User experience for teachers and students

One of the challenges for teachers of any new intervention is how to integrate it into their practices and curriculum, and how to get comfortable with the approach and feel confident in its use with students. MLM is partnering with Desmos[20], well known for its online graphing calculator, which is now embedded in many assessment systems, and for its activities that support students to create their mathematical ideas and share them with others. Recently Desmos has begun adapting the highly rated and openly licensed curriculum authored by Illustrative Mathematics (IM) in collaboration with Open Up Resources[21]. Through our partnership, all MLM teachers will have access to the Desmos version of the IM middle grades curricula (8th grade is ready and 7th grade will be soon, followed by 6th). This means that teachers will have readymade activities for any point in their curriculum that provide the enhanced experience of Desmos, where students see their ideas in context, get mathematical feedback that they can use to revise their thinking and build new ideas. In this way MLM is prepared to be responsive to how schools wish to incorporate MLM, including wholesale replacement of their existing curriculum. Subject to the co-design process, we expect many teachers to use MLM initially for unit launch activities, where the focus is on engaging students in the topic, activating their prior knowledge, and orienting them conceptually to the key ideas. Through the co-design process, MLM will identify three core math topics that will be the focus of MLM studies of impact and which we need a core set of participating classrooms to use. It is MLM's intention that there is an initial intensive period of use of MLM over a few weeks that orients students to the approach and brings them to the point where they are functioning well in the system. This, then makes it possible for teachers (and students!) to use other activities as these become appropriate.

Typically teachers will select an activity and ideally let the MLM development team know in advance so that we can customize it as needed and ensure that it is ready for use. MLM will have an online partnering schedule for those who want to form collaborative groups with other schools. The MLM program will be designed to work well for either within class or external collaborations. Students will do the activity, typically for a class period, sometimes two. The teacher reviews their sessions and selects several moments for the subsequent discussion to help students learn from each other's work and the teacher helps them consolidate resulting concepts and procedures. Periodically teachers facilitate student reflection on their work, picking one of the key categories of the project: math concepts or practices, collaborative practices/EFP, or equity. Students pick a snippet of their session and make a video (we are planning to use FlipGrid, at least in the early phases) of themselves talking about their journey as a mathematical thinker, which they then share with their collaborators or the whole MLM community. Students are encouraged to leave comments for at least two other students in response to their videos.

Ideally MLM is positioned as a resource that students initiate use of as they figure out how it serves them, rather than as another obligatory math exercise. Spontaneous problem solving and community activity is encouraged and celebrated. Students will use it to share and explore math and connect with other students. They will try to get good at collaborating in math problem solving. It should feel personal and perceived as a resource that supports their success and interests.

Barriers and mitigation:

Qualified Teachers - We know from experience and from feedback by the Educator Leadership Council that there are schools that need programs that work even when there is no teacher available who is prepared to provide a high quality math learning experience, not to mention prepared to use MLM well. As an online program, that will have a growing community of collaborating students, teachers, and schools, the opportunity will exist to partner well-resourced and under-resourced contexts.

Technology - There is no escaping that MLM is technology-based. It is possible to implement the collaborative problem-solving approach and the story telling without technology, to good effect. Nonetheless, there are many dimensions of MLM that require use of technology and networks and that are designed to significantly enhance the benefits.

Data overload - The fact that MLM enables access to the thinking process through review of sessions also poses some challenges. Reviewing student collaborative sessions can be time-consuming. We have previously collaborated with an Israeli group, under the direction of Baruch Schwarz that developed a dashboard (SAGLET)[22] that identifies critical moments of sharing, coordination, and meaning making which teachers can use to quickly learn how a group is doing and where to look for interesting activity. They have agreed to let us try to build this into the MLM environment.

Students at different levels - It is expected that participating schools will have classrooms of students at different levels. One of the benefits of the MLM system is that the math software, rich tasks, and collaborative learning provide lots of support for engagement and learning at all levels. The dynamic math environment enables a lot of learning through basic actions of dragging and trial and error, giving mathematical feedback as students manipulate objects on the screen. The activities are designed for low barriers of entry and high ceilings of learning. As the students get good at collaborating, and with effective groupings, everyone is able to make meaningful contributions and the esteem of low performing students is enhanced when engaging high quality tasks productively. As the curriculum and number of activities is built out, there will also be more opportunities for teachers to differentiate learning by the choice of activities for each group.

Responsive to each and every - Relevance is determined by the learner and it can appear to be a daunting challenge to be responsive to the interests and experiences of each and every student. Some interventions focus on topical relevance, providing content set in different real-world situations familiar to the learner, clearly an overwhelming task if one has to create such content, and, while there are many such curricular resources, there are not near the amount needed for all interests and all math topics. MLM will facilitate teachers' awareness of and access to the better materials available and will also have a Relevance Hotline where students and teachers submit ideas and questions about particular areas of interest that MLM can research and design for. However, responsive teaching focuses on more than the content of math activities. Responsive teachers are aware of and value the different experiences and understanding that students bring to classrooms. Such teachers seek to elicit more information from students about their thinking and their experiences, develop their cultural knowledge and use it to interpret student participation, and demonstrate belief in their students' capacity and respect for their ways of sense making and valuing in mathematics. The MLM program will provide professional development and resources, in part drawing on expertise in the participant community itself, that support all of us to increase our cultural proficiency and responsiveness.

Teacher time for planning, recording, participating in lesson study, and design meetings -We have used an online format involving 2-5 hours online over each week, each teacher participating when they are able, along with periodic live video conferences of 2 hours. In some projects our district partners have arranged for a course down during the intensive initial year of participation. We did not budget for teacher stipends for out of school time, understanding that there is financial support through EF+Math for its school partners.

Students with learning differences - the Desmos calculator, used in the online collaborative activities, is designed to support visually impaired students. We hope to make use of their expertise to build assistive text into the other aspects of the collaborative environment. Our project would benefit from additional help thinking more about learning differences and participation in the MLM.

Mathematics

The mathematics content of the MLM Project will consist of these three strands, each critical for middle school students' participation in higher mathematics and other STEM fields: rational numbers, algebra, and geometry. Knowledge of rational number, in particular, fractions is the key for students to succeed in Algebra and later mathematical disciplines[23242526] Algebra is replete with examples directly and indirectly related to fractions such as to understanding the behavior of linear function, solve quadratic equations by completing the square, resolve systems of linear equations, manipulate rational expressions, and reason with ratios as probabilities. A large basis of proportional and algebraic thinking rests on a clear understanding of rational numbers concepts and on the ability to operate and manipulate fractions[27282930] . Furthermore, the knowledge of fractional numbers and their behavior in proportions are fundamental for the understanding of advanced areas such as the theory of limits, calculus, numerical analysis, and real analysis.

Complementing their importance for students at school, fractional numbers influence their future in the job market and, in general, in social justice. Controlling IQ, education and family income, the knowledge of fractions by students predicts their future occupation and income[31]. So, this knowledge plays a significant role in issues of social equity. In several publications, the African American civil rights fighter and founder of the Algebra Project, Bob Moses and also with his colleagues[32333435] argue compellingly that access to algebra for African Americans and other students of color is part of the struggle for social justice. By extension, as the study of fractions precedes introduction to algebra and predicts performance in it, opportunities to learn both rational numbers and algebra are crucial for students of color to achieve social justice and equity.

The third content strand of MLM Project is geometry. The dynamic geometry environment within an online collaborative space is particularly well suited to involve students in enhancing their executive function in the practice of geometric problem solving and attending to the feedback of actions that transpire on their digital device (desktop, laptop, or tablet) as they and their collaborators move objects about to understand degrees of freedom and functional dependency. These were outcomes that we observed in studies involving teachers[3637] . The MLM dynamic geometry instructional materials develop students' awareness and implementation of mathematical practices[38] as a habit of mind[39] .

MLM will also provide activties for all of the grades 6-8 Desmos implementation of the Illustrative Mathematics curriculum.

Executive Function:

The key feature of our approach to executive functioning is our embedded view of the topic. Rather than train EF in isolation and hope for transfer to math, we envision our collaborative learning environment as the main engine of EF enhancement. The actions of collaboration will draw on and strengthen executive functions in practice (EFP). Specifically, we draw a connection between turn taking and inhibitory control, holding ideas of others in mind and working memory and incorporating other viewpoints into our own as evidence of cognitive flexibility. We will look for these EF practices by using content analysis of the online collaborative environment. We will pair these qualitative measures with more traditional measures of EF (see Measurement Grid). We contend that low EF often found in our target population results in part from lack of challenge in the educational settings. By providing students with rich complex math instruction and collaborative online environment for exploring this mathematical material, they are provided with ample opportunity to engage their executive functions in a meaningful context.

That said, we are mindful that the complexity of this approach can introduce additional cognitive load[40]. Thus, we balance the demands of collaboration with the simplicity of the VMT environment. Specifically, this interface minimized extraneous load by removing video or audio channels and provides just two input streams: the mathematical canvas and the chat window. A second relevant feature of our design is that only one student can control the mathematical canvas at a time. Students must request and be granted permission to their peers in order to express their ideas. We have found that this design feature contributes to building collaborative norms such as turn taking and considering the ideas of others, exactly the EFPs we expect MLM to foster.

Finally, we note that the vast majority of programs that specifically train EF have short training periods, usually less than 30 minutes per day[41] likely due to the boringness of most EF training activities. In contrast our VMT sessions take a typical class period (50 minutes) and can run up to 1.5 hours in schools using double period flex scheduling, thus providing much greater "time on task".

Executive Function and Mathematics:

The relationship between EF and academic achievement, especially mathematics, is well established, but leveraging this insight to improve academic outcomes remains elusive. The initial concept of EF, especially working memory, as a mental workspace[42] coupled with the advent of "brain training" studies reporting improvements in working memory[43][44] lead to the hypothesis that improving EF could improve math outcomes. Yet, this work has largely failed[45] suggesting that the relationship between EF and academic achievement is more complex than EF enables math and hence larger EF capacity will yield better math outcomes.

In fact, the relationship may be bi-directional as mathematical training can improve EF capacity, as seen in studies of abacus training[46], which improved performance on a Go/No-Go measure of inhibitory control. Moreover, simultaneous training both working memory and math skills is more effective for mathematical outcomes than training on either task in isolation[47]. Building on these results, we take the position that these two capacities are mutually reinforcing. We further qualify these relationships by noting that EF may be particularly important for the acquisition of new math knowledge[48]. This approach is also supported by prior work[49][50] suggesting that that oversimplifying instruction for underresourced learners can actually be counter-productive.

We also recognize that different math content areas may rely on different aspects of executive functions. For example, learning new material which contradicts prior knowledge as in the case of rational numbers, may particularly draw on inhibitory control[51][52]; whereas complex geometry problems may rely more on visual spatial working memory[53]. However, the separability of these constructs has been challenged[54] especially in children[55]. Our approach to EF in Practice means that multiple aspects of EF will be engaged regardless of the content domain, making us uncertain as to the specificity of our approach for building specific EF functions. Supplementary Study 2: Cognition and Neuroimaging is aimed, in part, at addressing this question by examining changes in brain activity for different executive functions (i.e. working memory and inhibitory control). Interestingly, intensive abacus training not only improved behavioral measures of inhibitory control in 4th and 6th graders, but also increased neural efficiency (reduced activity) during the Go/No-Go task. Thus, by including neuroimaing we can go beyond behavioral measures of transfer to examining the mechanism behind successful learning[56].

A final component to our approach to merging EF and mathematics is building meta-cognitive awareness in students of the role of EF in math learning and performance. Emerging work on EF and meta-cognition (MC) suggests they both contribute to math outcomes[57]. We will measure MC both in terms of individuals' general propensity towards reflection[58] and second in students understanding of EF in their own problem solving.

Outcomes

Through rapid cycle in-school testing in Phases 1 and 2 of the project will be used to adjust the design and implementation of the MLM. In these phases, we will address:

1. Students' participation in the MLM program and whether the student's engagement, triggers for interest, the co-design process, and implementation pacing and dosage are working as expected.
2. Students' math achievement, EF functioning, and equity (operationalized as their feelings of self-efficacy and belonging, interest in, and identification with mathematics) in relation to baseline data in order to determine MLM effects. Although previous studies do not suggest that we should see immediate results[14], analyses of data in each phase will inform the assessment and tracking of change, as well as understanding of the processes and development associated with MLM and their pacing.

In each cycle, we will assess change in relation to student factors such as: conceptual understanding of mathematics, EF, and equity. We will also examine factors related to teachers in order to understand variations in MLM implementation. These factors will include their investment, focus and development (e.g., in terms of pedagogical practices, support of students' mathematical thinking, EF, and equity); variance in implementation practices; and use of technology in the classroom.

In the planned studies, there are a number of questions to be examined. Two of the key questions informing project research include:

1. In what ways does MLM support students' stronger awareness of, as well as changes in, their use of executive functions in practice?
2. What are the relations among changes in EFP and changes in: conceptual understanding in mathematics, identities and interest in mathematics? What patterns in activity (and changes in activity) and for which students are associated with significant gains in conceptual understanding of mathematics, interest in, and self-efficacy for learning and problem solving?

Research Design

MLM is a complex system[59]. In order to consider the magnitude of MLM effect and to characterize the phenomenon, quantitative and qualitative data collections are planned.

We will begin rapid cycle project work by co-designing and developing the MLM environment with teachers and their students, adapting existing measures, and classroom and instructional artifacts to be used as data sources, and collecting baseline information on students. During the first two phases of the grant, we also will conduct quasi-experimental studies to gauge the effectiveness of MLM processes that provide and build on an understanding of participants' work with MLM in terms of its feasibility, usability, and early indicators of MLM's impact on EF development, conceptual understanding of mathematics,and equity, as well as the relation among these elements of the MLM.

In the third phase of the project we will continue to seek and use feedback to enhance students' and teachers' experience working with the MLM, and its scaling. We will also begin scientific research to establish that the MLM is a valid program for promoting positive change in students' conceptual understanding of mathematics, EF, and equity. In Phase 3 this involves studying the large-scale group of all students (approximately 1000 students) in the seventh and eighth grade classrooms of all teachers who have participated in the MLM Co-design Teacher Development Process and are using MLM in their classrooms, in addition to a representative group of control classrooms.

Additionally, beginning in Phase 3 and continuing into Phases 4 and 5, we will engage in more targeted research studies. In order to avoid research fatigue, two purposeful groups of the seventh grade students will be drawn from the large-scale group of students being studied, for one of three supplemental studies: (1) an in-depth, micro-genetic analysis using qualitative data (approximately three-to four classrooms of students, 100 students) collected in September/October, January/February, and April/May; and (2) a cognition and neuroimaging study neuroimaging study (100 students, cognitive battery; 60 of these students, neuroimaging study) who will be studied before and after working with MLM for one content domain (e.g. rational numbers or geometry) during the school year. To focus this research effort and proximity to facilities and expertise, study assignment for these additional studies will be at the level of the school (e.g., in a school district with 2 schools of students of similar demography, seventh grade students in one school will be studied qualitatively, students in the next school will complete a cognitive battery and participate in a study of neuroimaging).

All seventh grade students studied in Phase 3 will continue to be studied at the same intervals, longitudinally through the fourth and into the fifth phases of the project. This will allow assessment of MLM impact over time, and a comparison of MLM effects to those reported by Boaler and Staples[14] who found that by year 2 in their study, initial achievement gaps were closed. In addition, during Phase 4 we will study all seventh and eighth grade students of all additional teachers using MLM (1,000 students), as well as a representative group of control classrooms. Data will be collected in September and in March; this will allow study of replication, the impact of the MLM implementation, and identification of issues associated with scaling use of MLM.

Finally, in Phase 5 of the project, the large-scale and longitudinal components of the supplemental studies will be completed in May, 2023. Given sufficient funding, a randomized controlled study of 10,000 students will be conducted, as well. If funds do not permit this, then convenience sampling of additional schools will be studied.

Implications of research findings

MLM is expected to have policy implications for teacher training and mathematics education, and to extend present understanding of EF and practice. The MLM design draws on pedagogies that have been demonstrated to enable students to achieve and seriously engage mathematics learning: challenge based instruction can be expected to develop higher levels of conceptual knowledge and performance than possible with direct instruction[1660]; complex instruction should result in broadened conception of mathematics that enable equity and the closing of existing achievement gaps[1415]; and culturally relevant text should positively impact comprehension[61], as well as students' developing confidence in and interest for mathematics[62]. Whereas concern has been voiced that reform practices only benefit and increase privilege, MLM practices should provide clear evidence that underrepresented students who have previously had difficulties with math will successfully engage with rich math and come to recognize themselves as mathematical thinkers.

B. Rigorous R&D Approach to Equity, Privacy, and Co-Design

At one level, equity means achieving results such that no group of students, especially defined by categories such as race and ethnicity, look any different in their educational achievement and opportunities than any other such group. The work of equity is the work of humanizing math education: centering, valuing, being responsive to, and recognizing the authority of student (and teacher) voices as they express their interests, needs, understandings, thinking practices, and ways of engaging. It involves removing the barriers to participation, focusing resources of the highest quality that compensate for years of deficit, and recognizing and supporting the power of students of color (and their teachers) to shape a path to success in such a context. Co-design is an important component of this work in MLM.

The MLM design involves teachers and their students working with us to build capacity. We think of this as a collaborative, democratic process, where teachers and students are members of the team. We will involve teachers as participants in the MLM and provide information about the research questions we are examining and what we are learning in order to enlist their help in thinking about what really works.

Project plans include a virtual summer development institute where the stakeholders all work together to develop curricular tasks, scaffolding, and teacher professional development (pd). During piloting and implementation, the project design team, with several partner representatives, will meet regularly one or more times a month to review what has occurred, been learned and needs to be done. It is our experience through the Math Forum[63][64] that some of the participants will want to move into other leadership roles in the project such as conducting pd, community mentoring, and ongoing curriculum development. In addition, there will be a virtual advisory group that includes other teachers and school leaders to give us perspective and advice, making sure the project is moving in a direction that is generalizable to other settings and attending to all dimensions. We also imagine a “residency program” where at least one teacher would join the full-time staff for a semester or more each year.

In addition to this project-level, team-based process, participant-driven design for equity is also embedded in the core activities of MLM as described above, centering the student and teacher voices.

C. Risks and Mitigation

In addition to those noted above under Barriers and Mitigation concerning the MLM learning process, there are other project levels risks:

- Software and curricular development could lag behind teacher readiness and school calendars. Mitigation: we will be prepared to do collaborative learning the curricular materials without use of the enhanced online environment.
- It is proving difficult to coordinate collaborative sessions between schools in different locations. Mitigation: it might help to have selected locations in the same time zone. We can try variations that aren’t synchronous where teams work at different times, building on each other’s work rather than always working at the same time. The more different schools we have, the better our chances of finding matches. Nonetheless, the collaborative environment works well within a class and within a school.
- Teachers aren’t able to make enough time to get comfortable implementing the program in all its aspects. Mitigation: we can slow down the pace of activity, taking smaller steps, because we have an aggressive start, working with more classes early on than we need to hit the program targets.
- Some students are not functioning well in the online environment. Mitigation: teachers and project staff will work to anticipate this and incorporate mixed activities, physical andvirtual, that make use of best practices curriently working in those classrooms.

D. Data Rights

The content of the Desmos-IM materials will also be released under a Creative Commons BY or BY-NC license as appropriate. The technology and Desmos application are owned solely by Desmos and would not be transferable nor free to use by anyone else without explicit agreement by Desmos. If EF+Math exercises the license to the MLM program, Desmos would work with EF+Math in good faith to support the ongoing program under reasonable contractual terms that would at least enable Desmos to maintain the working connections on which MLM depends.

E. Discussion of Related Research

In an NSF-funded, seven-year project, "Computer-Supported Math Discourse among Teachers and Students," (DRL-1118888 and DRL-1344980), to integrate into an existing online portal—Virtual Math Teams (VMT)—with a dynamic math software—GeoGebra—to create a multi-user, synchronous math collaboration environment, and develop a curriculum and pedagogy for both teacher professional development and student learning. The project's salient findings included an understanding of how online, dynamic tools mediate students' mathematical activities and support them to build mathematical knowledge in a collaborative environment. The project also learned how students' establishment of collaborative norms support their appropriation of relations and properties of mathematics dynamic geometric objects[65][66] .

G. MLM Movie: http://files.mathematicalthinking.org/mlm/MLMTechVideo.mp4

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